

increasing_split^{4,23}

$$\begin{aligned} & \forall m:\mathbb{N}, P:(\mathbb{N}_{<m} \rightarrow \text{Prop}). \\ & (\forall i:\mathbb{N}_{<m}. \text{Dec}(P(i))) \\ \Rightarrow & (\exists n, k:\mathbb{N}, f:(\mathbb{N}_{<n} \rightarrow \mathbb{N}_{<m}), g:(\mathbb{N}_{<k} \rightarrow \mathbb{N}_{<m}). \\ & \text{increasing}(f;n) \\ & \& \text{increasing}(g;k) \\ & \& (\forall i:\mathbb{N}_{<n}. P(f(i))) \\ & \& (\forall j:\mathbb{N}_{<k}. \neg P(g(j))) \\ & \& (\forall i:\mathbb{N}_{<m}. (\exists j:\mathbb{N}_{<n}. i = f(j) \in \mathbb{Z}) \vee (\exists j:\mathbb{N}_{<k}. i = g(j) \in \mathbb{Z}))) \end{aligned}$$